

Name:

Solutions / Answers

1. You must show all the steps in your work. Find  $y'$  given  $y + \cos y = x$

$$\begin{aligned}y' + -\sin y \cdot y' &= 1 \\y'(1 - \sin y) &= 1 \\y' &= \frac{1}{1 - \sin y}\end{aligned}$$

2. You must show all the steps in your work. Find  $\frac{dy}{dx}$  given  $\frac{y^2}{2} + 2x - 4y = 1$

$$\begin{aligned}\frac{2y \cdot y'}{2} + 2 - 4y' &= 0 \\yy' - 4y' &= -2 \\y'(y - 4) &= -2 \\y' &= \frac{-2}{y - 4}\end{aligned}$$

3. You must show all the steps in your work. Find  $y'$  given  $x = \sin^2 y$

$$\begin{aligned}1 &= 2 \sin y \cdot \cos y \cdot y' \\\frac{1}{2 \sin y \cos y} &= y'\end{aligned}$$

4. Find  $\frac{dy}{dx}$  given  $x = y^2 + 4y - 6$

$$1 = 2y \cdot y' + 4y'$$

$$1 = y'(2y + 4)$$

$$\frac{1}{2y + 4} = y'$$

5. You must show all the steps in your work. Find  $y'$  and the slope of the curve of  $x^3 + y^3 = 7$  at the point  $(-1, 2)$

$$3x^2 + 3y^2 \cdot y' = 0$$

$$3y^2 y' = -3x^2$$

$$y' = \frac{-3x^2}{3y^2}$$

$$y' = \frac{-x^2}{y^2}$$

$$\text{at } (-1, 2)$$

$$y' = \frac{-(-1)^2}{2^2}$$

$$y' = -\frac{1}{4}$$

6. You must show all the steps in your work. Find  $y'$  given  $2x^{\frac{3}{2}} + x \tan y = y$

$$\frac{3}{2} \cdot 2x^{\frac{1}{2}} + \tan y + x \sec^2 y \cdot y' = y'$$

$$3\sqrt{x} + \tan y = y' - x \sec^2 y \cdot y'$$

$$3\sqrt{x} + \tan y = y'(1 - x \sec^2 y)$$

$$\frac{3\sqrt{x} + \tan y}{1 - x \sec^2 y} = y'$$

7. You must show all the steps in your work. Find an equation of the line that is tangent to the curve of  $2xy + \pi \sin y = 2\pi$  at the point  $\left(1, \frac{\pi}{2}\right)$

$$\begin{aligned}
 2y + 2xy' + \pi \cos y \cdot y' &= 0 \\
 y'(2x + \pi \cos y) &= -2y \\
 y' &= \frac{-2y}{2x + \pi \cos y} \\
 \text{at } \left(1, \frac{\pi}{2}\right) \quad y' &= \frac{-\pi}{2 + \pi \cos \frac{\pi}{2}} = \frac{-\pi}{2 + \pi(0)} = -\frac{\pi}{2} \\
 y - \frac{\pi}{2} &= -\frac{\pi}{2}(x - 1)
 \end{aligned}$$

8. You must show all the steps in your work. Find  $\frac{dy}{dx}$  given  $x^2 - xy + y^2 = 1$ .

a.  $\frac{2x+y}{x-2y}$

b.  $\frac{y+2x}{2y-x}$

c.  $\frac{2x}{x-2y}$

d.  $\frac{y-2x}{2y-x}$

e.  $\frac{y+2x}{x}$

f. None of these

$$\begin{aligned}
 2x - y - xy' + 2y \cdot y' &= 0 \\
 2y \cdot y' - xy' &= y - 2x \\
 y'(2y - x) &= y - 2x \\
 y' &= \frac{y - 2x}{2y - x}
 \end{aligned}$$

9. You must show all the steps in your work. Given the relation  $x^2 + y^2 = 64$ , then  $\frac{d^2y}{dx^2} =$

a.  $\frac{y^2 - x^2}{y^3}$

b.  $\frac{-y^2 + x^2}{y^3}$

c.  $\frac{y^2 + x^2}{y^3}$

d.  $\frac{-y^2 + x^2}{y^3}$

e.  $\frac{-y^2 - x^2}{y^2}$

f. None of these

$$\begin{aligned}
 2x + 2y \cdot y' &= 0 \\
 2y \cdot y' &= -2x \\
 y' &= \frac{-2x}{2y} \\
 y' &= \frac{-x}{y}
 \end{aligned}$$

$$\begin{aligned}
 y'' &= \frac{-1 \cdot y - (-x) y'}{y^2} \\
 y'' &= \frac{-y + x \left(-\frac{x}{y}\right)}{y^2} \\
 y'' &= \frac{-\frac{y^2}{y} - \frac{x^2}{y}}{y^2} \\
 y'' &= \frac{-y^2 - x^2}{y^3} = -\frac{y^2 + x^2}{y^3}
 \end{aligned}$$

Optional Extra Credit

You must show all the steps in your work. Given the relation  $x^2 - xy + y^2 = 1$ , then  $\frac{d^2y}{dx^2} =$

$$2x - y - xy' + 2yy' = 0$$

$$2yy' - xy' = y - 2x$$

$$y'(2y - x) = y - 2x$$

$$y' = \frac{y - 2x}{2y - x}$$

$$y'' = \frac{(y' - 2)(2y - x) - (y - 2x)(2y' - 1)}{(2y - x)^2}$$

$$y'' = \frac{2yy' - xy' - 4y + 2x - (2yy' - y - 4xy' + 2x)}{(2y - x)^2}$$

$$y'' = \frac{\cancel{2yy'} - xy' - 4y + \cancel{2x} - \cancel{2yy'} + y + 4xy' - \cancel{2x}}{(2y - x)^2}$$

$$y'' = \frac{3xy' - 3y}{(2y - x)^2}$$